UNIVERSITY OF KONSTANZ
Department of Physics
Thiago Lucena, Dr. Andrey Moskalenko

http://tinyurl.com/qo2019

Quantum Optics
Winter semester 2018/2019 - Exercise sheet 12
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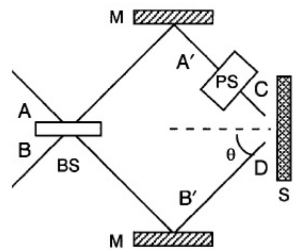
Problem 1: Quantum lithography.
Consider an experiment in which two beams, A and B, fall simultaneously into a 50 : 50 beam splitter (BS), and then are reflected by two mirrors, M_1 and M_2, into a screen S. The outcome is a well-known interference pattern. If a phase shifter (PS) is placed half-way between one of the mirrors and the screen, the interference pattern can be shifted along the length of the screen (x axis) by the PS. Independently of which kind of pattern is being formed on the screen, the use of classical light limits the maximally achievable resolution according to the Rayleigh criterion: $\Delta x_{\text{MIN}} = \lambda/2$. This limits the level of detail achievable with classical lithography.



a) Considering each of the processes previously described as an unitary transformation acting on the states of light, find the final state (before being absorbed by S) when the two input states are monochromatic and coherent (classical light), $|\alpha\rangle_A|\beta\rangle_B$. In the case when those states are identical and the two beams arrive at the screen from opposite directions tangentially to its surface ($\theta \to \pi/2$), show that the intensity profile is modulated as $1 + \cos(2kx + \phi_0)$, where ϕ_0 is the phase given by the phase shifter. What is the resolution of the formed pattern? HINT: remember that reflexion results in a phase change by a factor of π. The resolution depends on the periodicity of the patterns.

b) Consider now that the transformations should be applied on the field mode operators, $\hat{a}$ and $\hat{b}$, instead of the states. For this purpose work with the vector $[\hat{a}, \hat{b}]^T$. What are the matrices that describe each of the involved steps? Using these matrices, show that the final mode operators $\hat{a}'$ and $\hat{b}'$ (describing the states right before the light falls on the screen) are given by:

$$\hat{a}' = -\frac{e^{i\phi}}{\sqrt{2}}(\hat{a} - i\hat{b}), \quad \hat{b}' = -\frac{1}{\sqrt{2}}(\hat{b} - i\hat{a}).$$

HINT: for the PS transformation, use the phase $\phi = 2kx + \phi_0$.

c) The absorption of N photons on the screen results in a pattern modulated by $\langle\psi_0|\hat{e}^{\dagger N}\hat{e}^N|\psi_0\rangle/N!$, where $\hat{e} = \hat{a}' + \hat{b}'$ and $|\psi_0\rangle$ is the input state. Using the two-photon absorption pattern ($N = 2$) as criterion, show that the resolution obtained for a two-photon input state of the same frequency $|1\rangle_A|1\rangle_B$ is better than the one obtained for classical input states $|\alpha\rangle_A|0\rangle_B$ and $|\alpha\rangle_A|\beta\rangle_B$. This is the principle behind quantum lithography.

d) It can be proven that for a NOON state, $|\psi_{\text{NOON}}\rangle = (|N\rangle|0\rangle + e^{iN\phi}|0\rangle|N\rangle)/\sqrt{2}$, the N-photon absorption pattern reads $\langle\psi_{\text{NOON}}|\hat{e}^{\dagger N}\hat{e}^N|\psi_{\text{NOON}}\rangle/N! = 1 + \cos(N\phi)$. What is the maximum resolution of it? If a surface pattern of the form $p(\phi) = 1 - \Theta(\phi - \pi/2) + \Theta(\phi - 3\pi/2)$ for $\phi \in [0, 2\pi]$ and $p(\phi + 2\pi) = p(\phi)$ ($\Theta(\phi)$ is the Heaviside function) is desired, is it achievable by using a superposition of NOON states? What other states you think could be used to attain such a pattern?